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#### **Chapter 1: Introduction to** *The Solutions Project*

#### **Overview**

*The Solutions Project* is a research-based mathematics curriculum designed to teach arithmetic problem solving to elementary and middle school students with severe disabilities and/or autism. Students with disabilities who are also English Language Learners may benefit from the supports included as well. The curriculum addresses the adaptations required for instruction of students who are nonverbal, have visual impairments, have physical limitations, or who are deaf. The curriculum was designed for students with solid early numeracy skills, who are ready for the next level of mathematics. *The Solutions Project* provides instruction on solving real-world problems using addition and subtraction across a variety of formats including paper-and-pencil, SMARTBoard, and video simulation problems. *The Solutions Project* is comprised of six units with multiple theme-based lessons per unit. Each lesson is scripted, making it teacher friendly and easy to use. It also provides scripted praise, prompting, and error correction procedures. Three problem types are addressed in this curriculum, including the *group, change*, and *compare* problem types.

The intervention incorporates schema-based instruction combined with evidence-based practices for teaching academics to this population, and includes technology supports and self-monitoring. The purpose is to teach students to recognize underlying problem structures in word problems for better generalizability to real-world situations. This manual is laid out in the following sequence: (a) discussing research that has emerged on teaching mathematical problem solving, (b) the scope and sequence, (c) the components and instructional strategies included in the curriculum, (d) what to teach and how to create mathematical problems for SSD to be successful, and (e) how to teach the student to be a mathematical problem solver.

# **Chapter 1: Research Foundation**

#### **Rationale for Content and Method of Instruction**

The ability to apply mathematical concepts and problem solve are an integral part of everyday life. Problem solving has been deemed the cornerstone of mathematical learning (NCTM, 2000) and is a critical skill for being able to function in the 21<sup>st</sup> century. The Common Core State Standards have placed great emphasis on problem solving and it is a standard for mathematical practice. The ability to learn mathematical word problem solving translates to better real-world problem solving.

Problem solving is a complex skill that requires higher order thinking. Kearns, Towles-Reeves, Kleinert, Kleinert, and Kleine-Kracht Thomas (2011) found in a sample of 12,649 students who took alternate assessments based on alternate achievement standards across seven states, only a small percentage (4-8%) of students were able to apply computational procedures to solve real-world or routine word problems from a variety of contexts. There is a need to improve mathematical problem solving for students with intellectual disabilities using empirically based methods.

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# **Schema-based Instruction**

Schema-based instruction (SBI) uses a conceptual teaching approach which combines mathematical problem solving and reading comprehension strategies (Jitendra, 2008). SBI focuses on conceptual knowledge by enhancing comprehension to ensure students can effectively create representations of the problem situation, thus developing an understanding of the underlying problem structure. This step is imperative to successful problem solving because most errors in word problem solving are actually a result of students misunderstanding the problem situation, rather

# Conceptual Comprehending Modeling the problem

**Effective Problem Solvers Combine:** 

than computation errors (De Corte & Verschaffel, 1981). In SBI, students learn to understand the semantic structure of word problems through text analysis in order to identify quantitative relations between sets or actions between sets, and then learn to create a visual model of these relationships (Jaspers & Van Lieshout, 1994). From this mathematical representation, or model, students can select the operation to solve. The procedural rules for solving problem types are directly related to the underlying concepts. For example, rather than just teaching students to add when the total is unknown (i.e., the procedural rule), SBI would teach a rule that relates the concept to the algorithmic procedure (e.g., two small parts are combined to create a whole, or "part-part-whole;" Jitendra, 2008).

**Problem type and schematic diagrams.** There are three main types of arithmetic word problem situations (i.e., schemata) that have been identified; these are *group*, *change*, and *compare* (Marshall, 1990). In SBI, students are explicitly taught to identify the structural features of the problem (e.g., part-part-whole) in order to identify the problem schemata (e.g., group problem). Visual representations go far beyond simply a pictorial representation of the information in the word problem. These representations, called schematic diagrams, provide students with a way to visually organize and summarize the information from the word problem so that it is concrete and shows the relationship among numbers in the problem. Schematic diagrams help students develop a deep understanding of the problem and aid in the transfer of learning to novel problems (Zahner & Corter, 2010). Known values in the word problem are written into their corresponding parts of the selected schematic diagram, and the unknown value are either left blank or a question mark is placed in the diagram to indicate the unknown.

Group schema (Figure 2) involves two or more small groups combined to make a larger group, emphasizing the part-partwhole relationship. For example, *Sarah earned 3 tickets at the carnival. Jose earned 5 tickets. How many tickets did they earn?* Three and five tickets represent the "part" relationships and the unknown quantity (i.e., tickets in all) represents the "whole."

Part	Part
A	AII

Figure 2. Group diagram adapted from Willis and Fuson (1988).

Change schema (Figure 3) involves a dynamic process, where an initial quantity is either increased or decreased over time to result in a final quantity. An example of a change increase problem is shown below: *Marcus had \$2 from doing chores. He earned \$5 more doing chores. How much money does Marcus have now?* An example of a change decrease problem is shown below: *Marcus raked 5 piles of leaves. He bagged 2 piles of leaves. How many more bags does he have left to bag?* In both examples, Marcus starts with an initial set (\$2 and 5 piles). The initial set was changed by adding more or taking away from the set (\$5 and 2 piles), resulting in a final amount which is unknown. The change decrease problem also illustrates one issue that can result in incorrect operation when using the key word strategy. Specifically, the question uses both key words "more" and "left," which usually denote addition and subtraction, respectively; however, the problem requires the student to subtract to solve.



Figure 3. Adapted from Jitendra and Hoff (1996).

The compare schema (Figure 4) involves comparing two differing sets that are related in some way, and requires finding the difference between the sets, regardless of whether the question is asking "how many more" or "how many fewer." Here is an example: *Sam ordered 5 chicken nuggets. His brother, Drew, ordered 10 chicken nuggets. How many more nuggets did Drew order than Sam?* The two numbers being compared are the number of chicken nuggets Sam ordered (5) and Drew ordered (10) and the relation between these two sets is the difference of 5.



Figure 4. Adapted from Willis and Fuson (1998).

All of these examples displayed the unknown quantity in the final position of a number sentence. More complex problems may place the unknown in the initial or medial position, or include extraneous information in the word problem.

Why not just use the key word strategy or a mnemonic strategy (e.g., STAR)?

Keyword strategy. In modified schema-based instruction used in The Solutions Project, students learn to focus on the

problem structure before solving the problem. By doing so, students can differentiate between problem types and solve a variety of

problems. When students are taught the key word strategy, they are often set up for failure for numerous reasons.

- Many words can indicate more than one operation. For example, many students are taught "more" means to add, but this can be misleading. Take a look at a few examples.
  - In the first problem a compare problem the keyword "more" appears, but the operation used to solve this problem is subtraction, so if students were relying on the keyword strategy, they would be incorrect.
  - In the second problem a change problem the keyword "more" appears again, and like the prior example, this problem is solved by subtraction. This is a very applicable problem students may encounter in their daily lives, and it is important for them to be able to recognize the problem type and solve using the correct operation.
  - In the third problem a group problem the keyword "more" appears, and it is an addition problem; however, it is a two-step problem requiring the student to add "1+3" and then add the sum of that number to "3" resulting in a total sum of seven. Although two-step problems are not used in *The Solutions Project*, this illustrates another error that would result if students were simply taught "more" means to add.



• Many problems do not contain key words in them (see example below). When thinking about mathematical problems in real-life, there will be no key words for students to identify. They have to understand the problem, identify how to solve it operationally based on what is happening, and then carry out the solution. With generalizability to solving real-world problems as one of the most important goals for individuals with disabilities, teaching students to understand the structure of the problem so they are better able to generalize to real-world problem solving is of utmost importance.



• As illustrated in all of these problems, keywords can be a part of word problems, but they do not always reflect the problem type. Keywords do appear in many of the problems in *The Solutions Project*, but students are taught to identify the problem type through structural features rather than relying on keywords.

Mnemonic. When considering the population of students using *The Solutions Project*, many may be non-readers, emerging

readers, or may have memory deficits. A mnemonic like "STAR" which stands for search, translate, answer, and review, may not be

helpful. Students may not be able to remember what the letters stand for, and most students needs the problem solving process broken down into further steps. *The Solutions Project* uses a student-friendly task analysis with picture prompts to help them solve the problems.

Key components of SBI. SBI has four main components, including: (a) identifying the underlying problem structure, using visual representations known as schematic diagrams; (b) explicitly teaching problem solving through the use of a heuristic (a problem solving strategy, frequently a mnemonic); (c) using explicit instruction to teach the four-step problem-solving heuristic (i.e., problem schema identification, representation, planning, and solution); and (d) metacognitive strategy knowledge instruction, which includes activities such as analyzing the problem, self-monitoring of strategy use, and checking the outcome for accuracy. Visual representations help students organize key information from the problem. Students can be taught to organize information using a schematic diagram and show their solution using a mathematical equation (Griffin & Jitendra, 2009). Heuristics are explained at the end of the paragraph as they serve a dual purpose as a metacognitive strategy. Explicit, teacher-delivered instruction is essential to SBI. Initially, teachers model problem solving by demonstrating how to analyze text in the word problem to find key information and represent it in schematic diagrams. Rules and procedures are explicitly explained. According to Jitendra (2008), in SBI, students are taught to identify the problem type first and fill in the corresponding schematic diagram, using story situations with all known values. This is known as schema induction. The purpose of doing so is to teach students to analyze the story situation structure rather than acting impulsively, selecting numbers, and computing (Jitendra, 2008). Once students have shown mastery identifying the problem type, selecting the corresponding schematic diagram, and filling in the known and unknown values, they are taught to solve the The research reported here was supported by the Institute of Education Sciences, U.S. Department of Education, through Grant R324A130001 to the

problem. They are also taught to represent the schema in an equation (also referred to frequently as a number sentence in SBI). Students practice these skills repeatedly within each problem type, as well as with mixed problem types once they have learned all types. For metacognitive strategy knowledge, students are taught to use think-alouds to explain their reasoning (e.g., "Why is this a *change* problem?" It is a *change* problem because there was an initial set, a change set, and an ending set.). In addition, students are given a four-step strategy checklist with the heuristic to help transition from teacher-led instruction to student-led instruction. Jitendra et al. (2007) uses a specific heuristic of *FOPS* (e.g., F: Find the problem type, O: Organize the information in the problem using the schematic diagram, P: Plan to solve the problem, and S: Solve the problem). Rockwell, Griffin, and Jones (2011) used a shortened version of Jitendra's heuristic, *RUNS* (e.g., R: Read the problem, U: Use a diagram, N: Number sentence, and S: State the answer) with a student with autism and mild ID.

#### **Effectiveness of SBI**

Numerous studies have shown that students with learning disabilities can be taught problem solving using SBI both at the elementary level and at the middle school level (Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra, Hoff, & Beck, 1999; Xin, Jitendra, & Deatline-Buchman, 2005; Xin & Zhang, 2009). Some research has also shown that the use of schematic diagrams can be faded, so students gradually transfer their skills of using a schematic diagram to using mathematical equations to represent the structure of a word problem, known as schema-broadening instruction (Fuchs et al., 2009; Fuchs et al., 2008). Xin, Wiles, and Lin (2008) further built on SBI by using word problem story grammar to enhance problem solving in elementary students with math difficulties.

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Although the research on SBI is very promising for students with learning disabilities and students who are at risk for mathematical disabilities, very little research exists on using SBI with students with intellectual disabilities, including students who also have ASD. The following sections will provide reviews of the available literature on SBI for students with intellectual disabilities.

SBI for students with ASD. Rockwell and colleagues (2011) examined the effects of SBI to teach all three types of addition and subtraction word problems (i.e., combine, change, and group) to one female student with autism and mild ID. Two dependent variables were measured: practice sheets which were used as a formative assessment during training phases, and problem-solving probes which were used to measure treatment effects. Scripted lessons were used for each instructional session. First, students were taught a four-step heuristic for problem solving, "RUNS," which stood for (a) read the problem, (b) use a diagram, (c) number sentence, and (d) state the answer. Next, each problem type was introduced one by one. The student was shown a story problem with all quantities known to facilitate schema induction. Direct instruction (i.e., teacher modeling, guided practice, independent practice, and continual feedback) was used to teach the salient features of word problems. Then the student was asked to sort the problems into categories as belonging or not belonging to the type being taught. Once the student was able to discriminate problem type, she was taught to solve the problems of that type where the final quantity was the unknown. During generalization, an instructional session was given on using algebraic reasoning to solve problems of any problem type with unknown quantities in the initial or medial position. The student was able to master all problem types and generalize two of three problem types.

Rockwell (2012) replicated her 2011 study (Rockwell et al., 2011) with one 7-year-old male and one 12-year-old male with ASD. Neither student had a reported IQ score; however, both students attended general education mathematics classes with the The research reported here was supported by the Institute of Education Sciences, U.S. Department of Education, through Grant R324A130001 to the University of North Carolina at Charlotte. The opinions expressed are those of the authors and do not represent views of the Institute or the U.S. Department of Education.

support of a paraprofessional. In this study, Rockwell added one generalization measure, which included irrelevant information in the generalization probe word problems. Both students achieved perfect scores for all three problem types. One student continued to meet ceiling for generalization and maintenance probes. The other student met ceiling for generalization but at the 8-week follow-up maintenance probe, his data dropped slightly but still remained above pre-generalization training performance.

Both studies expanded the SBI research by including students with ASD and showed that students with ASD can master problem solving, maintain it, and generalize to novel problem types through SBI. However, the students with ASD in both studies all had relatively higher intellectual disability.

SBI for students with moderate/severe disabilities. Only one study prior to The Solutions Project used SBI to teach problem solving to a student with moderate ID. Neef and colleagues (2003) taught a 19-year-old man with an IQ of 46 to solve *change* problems by teaching precurrent behaviors. Four different training phases were conducted in which the student was taught to identify precurrent behaviors (i.e., component parts of the word problem) including the initial set, the change set, key words to identify the operation, and the resulting set. The unknown for each problem could be in any location (i.e., the initial set, the change set, or the resulting set). Within each training phase, supports and prompts were strategically faded. First, the targeted component was always known and the researcher prompted student responses. Next, the targeted component was randomly known in approximately half of the problems and the researcher continued to prompt responses. Finally, the targeted component was randomly unknown and no prompts were provided. One schematic diagram was used for all problems. Probes consisted of 10 word problems and were given after the student reached 100% correct responses for two consecutive sessions under the unprompted condition. The results showed that The research reported here was supported by the Institute of Education Sciences, U.S. Department of Education, through Grant R324A130001 to the University of North Carolina at Charlotte. The opinions expressed are those of the authors and do not represent views of the Institute or the U.S. Department of Education.

teaching precurrent behaviors was successful in yielding accurate problem solving. There are several important aspects to note in this study that offer implications and needed adaptations in SBI for students with more severe disabilities. First, in this study, traditional SBI was broken into much smaller steps by teaching precurrent behaviors to a student with moderate ID across four training phases. Second, it took the student several sessions to learn to solve one problem type (i.e., 26 for initial set, 35 for change set, 17 for operation, and 2 for resulting set). Third, a strategic prompt fading procedure was used in each phase to aid in the transfer from teacher-led instruction to student-led instruction. Finally, all problems were read aloud to the student.

The study by Neef et al. (2003) offers some directions in teaching word problem solving through SBI with adaptions to students with moderate to severe disabilities. Additional strategies may be needed to support students with ASD and severe disabilities. First, problems will need to be simplified in reading level and extraneous information will need to be removed to accommodate the difficulty in reading comprehension of students with ASD and severe disabilities. Read alouds offer necessary accommodation to support students with ASD and severe disabilities, especially those who are non-readers (Browder, Trela, & Jimenez, 2007; Neef et al., 2003). Second, some precurrent skills will need to be taught (Neef et al., 2003). Third, manipulatives may be needed to represent the problem for students who lack fact recall (Bouck, Satsangi, Taber, Doughty, & Courtney, 2013). Task analytic instruction with system of least prompts has been found to be effective at teaching mathematics to students with ASD and severe disabilities and can be incorporated into SBI (Browder, Spooner, Ahlgrim-Delzell, Harris, & Wakeman, 2008). Rockwell et al. (2011, 2013) and Neef et al. (2003) paved the way for teaching problem solving with students with ASD and severe disabilities, respectively, but much more research is needed with this population.

Modified schema-based instruction (MSBI), used in *The Solutions Project* curriculum, was developed by PI, Dr. Diane Browder, and special education experts, Drs. Alicia Saunders and Jenny Root. Schema-based instruction experts (Asha Jitendra & Nancy Neef) and an elementary mathematics expert (Drew Polly) aided in the development and validation of the curriculum. The research team, co-PIs Dr. Ya-yu Lo and Fred Spooner, and researchers Dr. Luann Ley Davis and Chelsi Brosh, M.Ed., were integral in the planning and implementation of the research on MSBI.

#### Chapter 2: What is The Solutions Project?

*The Solutions Project* is designed to teach problem solving to students with moderate and severe disabilities in an explicit and systematic manner. Students with moderate and severe disabilities often lack mathematical problem solving skills, or have been taught ineffective methods, such as the key word strategy. Because problem solving is needed across all domains of mathematics in the Common Core State Standards, as well as in functional applications of problem solving, these deficits can interfere with a student's ability to access the general curriculum and solve real-world mathematical problems. The purpose of *The Solutions Project* is to build mathematical problem solving skills concurrently while providing grade-aligned mathematics. In order for the mathematical problem solving skills to generalize to real-world scenarios and situations, it is of utmost importance to practice the skills in different contexts, environments, with different materials and platforms, and possibly with different instructors.

*The Solutions Project* curriculum is designed to be taught in small groups of two to four students for ease of instruction and practicality, but can be taught individually as well. This curriculum was developed by conducting research in schools by teachers of students with moderate and severe disabilities and Autism Spectrum Disorder (ASD).

*The Solutions Project* includes six units of instruction with over 500 word problems presented in various formats, including paperand-pencil, SMARTBoard, and video simulation problems.

# The Solutions Project Components

*The Solutions Project* curriculum combines principles of schema-based instruction with evidence-based practices for teaching students with moderate and severe disabilities (Browder et al., 2008; Spooner, Root, Saunders, & Browder, 2017), and is comprised of four components (see Figure 5). Each component is built upon research-based strategies for learners with disabilities and include: (a) creating access to the problem for non-readers or emerging readers by using read alouds; (b) teaching students to conceptually understand the problem types and structures by mapping the story grammar, using graphic organizers, and representing the problems with concrete or virtual manipulatives; (c) teaching students to procedurally solve the problem using task analytic instruction and self-monitoring strategies; and (d) teaching them to generalize in multiple ways to promote real-world problem solving. To develop an effective method for teaching mathematical problem solving in a highly

structured manner, *The Solutions Project* curriculum is based on principles of direct and systematic instruction.



Figure 5. Logic model for teaching arithmetic problem solving to students with SSD.

# The Scope and Sequence of The Solutions Project Across Problem Types

The following table breaks down each problem type and the discrimination process used in *The Solutions Project*. The table includes a description of each, the operation used to solve, the graphic organizer and rule for each problem type and the discrimination between problem types chart, the procedure used to solve each, the early numeracy skills used in each, and the mathematical language used in each.

Problem	Group	Compare	Change-Add &	All
Туре			Change Subtract	
Problem Type Description	Group problems combine two or more distinct groups into one large group. The two or more distinct groups in the problem must be related (e.g., apples, bananas, fruits; the large group, fruit, is the related category). This problem type targets the part-whole relationship and is static.	Comparison problems have things/people with contrasting quantities and the difference is found. These problems involve two people or objects comparing quantities of one thing (e.g., John and Sarah compare amount of stickers each has) or one person comparing quantities of two things (e.g., Alicia compares the number of stickers to stamps).	Change problems are a dynamic problem type. The entire problem is about one thing. The starting amount of that thing is either increased (+) or decreased (-) based on the action occurring in the problem resulting in a changed ending quantity.	• Discriminate between group, change-addition, change- subtraction, and compare problem types
Operation	Addition only (+)	Subtraction only (-)	Either addition or	Either addition or subtraction (+

			subtraction (+ or -)	or -)	
Schematic Diagram	*small group, BIG GROUP'	*BIGGER number, SMALLER number, DIFFERENCE between the two"	"One thing, SAME, old to it or toke every, CHANGE"	Group       Compare         Compare       Change	
Rule to Remember Schematic Diagram	"small group, small group, BIG GROUP"	"compare (more/fewer), bigger number, smaller number, difference between the two"	"1 thing, same thing, add to it or take away, change"	See individual columns	
Solving Procedure using Schematic Diagram	1. Create sets in circles representing two small groups of differing things 2. Combine sets into the big group and count to solve	<ol> <li>Create sets in arrays: larger number in top array, smaller number in bottom array</li> <li>Find the difference between the two arrays by dragging quantity to "difference" circle and count to solve</li> </ol>	<ol> <li>Create starting amount set in oval</li> <li>Add more to the set for additive change action or remove quantity from starting set to trashcan for subtraction change action</li> <li>Drag total or remaining counters to end square and count to solve</li> </ol>	<ol> <li>Determine problem type and select corresponding schematic diagram</li> <li>Solve using procedures described under each problem type</li> <li>Sort problem on chart by problem type</li> </ol>	
Early Numeracy Skills Targeted	<ul> <li>Number ID to 10</li> <li>Symbol recognition (+) (=)</li> <li>Counting with 1:1 correspondence</li> <li>Creating sets of up to 9 objects</li> </ul>	<ul> <li>Number ID to 10</li> <li>Symbol recognition <ul> <li>(-) (=)</li> </ul> </li> <li>Counting with 1:1 <ul> <li>correspondence</li> </ul> </li> <li>Creating sets of up to <ul> <li>10 objects using</li> </ul> </li> </ul>	<ul> <li>Number ID to 10</li> <li>Symbol recognition <ul> <li>(+) (-) (=)</li> </ul> </li> <li>Counting with 1:1 <ul> <li>correspondence</li> </ul> </li> <li>Creating sets of up to <ul> <li>10 objects</li> </ul> </li> </ul>	See individual columns	

	<ul> <li>Combining sets to add up to 10 objects</li> <li>Creating addition number sentence</li> </ul>	<ul> <li>arrays</li> <li>Concepts of larger/bigger and smaller</li> <li>Concepts of more and less/fewer</li> <li>Finding difference between two sets</li> <li>Creating subtraction number sentence</li> </ul>	<ul> <li>Composing and decomposing sets</li> <li>Creating addition and subtraction number sentences</li> </ul>	
Mathematic al Language	<ul> <li>Common addition vocabulary: add, join, group, combine</li> </ul>	<ul> <li>Comparative words: "more than," "fewer than," less than"</li> <li>Common subtraction vocabulary: difference</li> </ul>	<ul> <li>Common addition vocabulary: add, plus, more</li> <li>Common subtraction vocabulary: minus, take away</li> <li>Action verbs that show adding more/or taking away: adds more, gets more, buys, sells, eats, finds, gives away, breaks, makes, pays, puts away</li> </ul>	

#### Chapter 3: How to Teach The Solutions Project Curriculum

#### Units

There are six units of instruction in *The Solutions Project* Curriculum. The first unit reviews early numeracy skills to get students up to speed and be most successful by building critical pre-skills and concepts prior to starting problem solving. This can help reduce cognitive demands for students later on in the curriculum when they are also learning to solve and discriminate problems. The skills within Unit 1 are reviewed and taught in a massed trial format (see Appendix A for skills assessed and data sheet). The remaining units target teaching one problem type to mastery, or discriminating between problem types once solving two or more problem types were mastered (Unit 2: Group Problem Type, Unit 3: Compare Problem Type, Unit 4: Discrimination between Group and Compare, Unit 5: Change-Addition, Change-Subtraction, Change- Mixed Addition and Subtraction, Unit 6: Discrimination between All Problem Types). Each problem type and discrimination between problem types are taught in a model-lead-test format. First, the teacher models the procedures for solving for a few days while the students respond to each step following her model. Then, during the "lead" phase of instruction, the teacher assistance is faded and the student is given a chance to respond independently while the teacher provides least intrusive prompting, error correction, and reinforcement. Finally, once the student has mastered the unit, the teacher will administer an assessment for the "test" phase of instruction to see how much the student can achieve without prompting and feedback, similar to a traditional assessment. It is important to move your students towards independence, so we advise progressing through the model lesson quickly (1-2 days) so the student can practice the strategy with the teacher providing least intrusive prompting.

#### **Evidence-based and Research-based Instructional Strategies**

*The Solutions Project* was constructed using sound instructional practices that are described in this manual. The curriculum combines the principles of explicit and systematic instruction into the curriculum by teaching using a model-lead-test procedure, discrimination training for problem types, embedding the system of least intrusive prompts within the task analysis, and using the constant time-delay procedure to pre-teach skills. This section provides an overview of the primary instructional strategies used in the curriculum that were derived from research.

#### **Task Analytic Instruction**

The complex skill of mathematical problem solving was taught through task analytic instruction. Steps for solving the word problems were broken down into 12 sequential steps, which were carefully refined over the many iterations of *The Solutions Project* in research. The same 12 steps are used to solve all three problem types- *group, compare*, and *change*. These 12 steps are taught through total task chaining, meaning the teacher models each step of the task analysis, followed by a chance for the students to immediately practice each step, and this process continues until all steps of the task analysis have been completed and the solution is found. The first six steps target identifying the problem type, which addresses teaching conceptual knowledge, and the last six steps target solving the problem by finding the solution, which addresses teaching procedural knowledge. In the first six steps, students are taught to identify the problem type using story grammar instruction and story mapping, so they learn the salient structural features of each problem type.

In *The Solutions Project*, the task analysis is put in a student friendly version known as the Problem Solving Checklist, which allows for students to self-monitor and manage their progress in problem solving. Once a step is completed, students are taught to

check it off on the Problem Solving Checklist. Pictures are paired with each step of the task analysis to support non-readers or emerging readers. When first learning the Problem Solving Checklist, students may ask for the steps to be read aloud, but eventually as students become more proficient with identifying what each step means, either by linking it to the picture prompt or by memory, the teacher should fade his/her support so students become totally independent in problem solving.

# **Read Alouds**

A read aloud is an evidence-based practice for improving comprehension skills in students with severe disabilities. A read aloud permits students who are non-readers or who perform significantly below level in reading or comprehension, to request a reader, such as a teacher, peer tutor, paraprofessional, or parent, to read the text aloud to them in order to provide access to the text. In *The Solutions Project*, students may request the problem be read aloud to them or individual steps of the task analysis, until proficiency is achieved as stated earlier. Requesting the text be read aloud is an important self-advocacy skill for students to learn, and students are encouraged in *The Solutions Project* to request all read alouds. Students who are nonverbal or with limited verbal abilities can be provided with a communication card with icons for "read please," "help," and "I'm finished." For students who may be readers but lack comprehension skills, teachers are encouraged to allow them to attempt to read the problem aloud first independently, followed by a re-read of the word problem by the instructor.

#### **Explicit Instruction**

Explicit instruction is an effective and efficient strategy for developing problem solving skills in students with disabilities. Explicit instruction within a mathematical task has three components: (a) the teacher demonstrates a step-by-step plan (strategy) for The research reported here was supported by the Institute of Education Sciences, U.S. Department of Education, through Grant R324A130001 to the University of North Carolina at Charlotte. The opinions expressed are those of the authors and do not represent views of the Institute or the U.S. Department of Education.

<sup>23</sup> 

solving the problem, (b) this step-by-step plan is specific for a set of problems, and (c) students practice using the same procedure/steps demonstrated by the teacher to solve the problem (Gersten et al., 2009). Because problem solving is broken down in a step-by-step format, the teacher can monitor student progress and provide feedback or error correction immediately on each step of the chained task thus resulting in a high rate of student success during practice. Once students are firm in their responding to steps of the chained task, teacher feedback is gradually and systematically faded during instruction and the student is expected to perform all steps in a problem independently (Darch, Carnine, & Gersten, 1984). Explicit instruction has recently been identified as an evidence-based practice for students with severe disabilities (Spooner, Root, et al., 2017), but has been well established as an evidence-based practice for students with learning disabilities (Gersten et al., 2009).

Explicit instruction consists of the instructor's modeling the targeted behaviors in a step-by-step format, guided practice where the student received an opportunity to respond first with a set amount of wait time before the teacher provides a prompt, and independent practice where the student gets to try to solve the problem independently with no assistance from the teacher unless the student requests something read aloud., and continuous feedback on skill performance (Stein et al., 2006). We followed this "model-lead-test" procedure with reinforcement and immediate error correction.

**Discrimination Training.** A form of explicit instruction known as discrimination training also was incorporated as a component of the instruction to teach students to decide when to and when not to apply the strategy (i.e., determine the problem type and decide when to add or subtract). This is done by teaching students to locate salient features in the word problem and then sort them by type in Units 4 – Discrimination between the Group and Compare Problem Type, and Units 6 – Discrimination between the The research reported here was supported by the Institute of Education Sciences, U.S. Department of Education, through Grant R324A130001 to the University of North Carolina at Charlotte. The opinions expressed are those of the authors and do not represent views of the Institute or the U.S. Department of Education.



Group, Compare, and Change Problem Types. In Unit 5, students also are taught to discriminate between change-addition and changesubtraction problems once they have mastered change-addition and change-subtraction problems individually first.

# **Using Time Delay**

The time-delay procedure is an evidence-based practice for teaching mathematical content to students with severe disabilities (Browder et al., 2008; Spooner, Knight, Browder, Jimenez, & DiBiase, 2011). Constant time delay is a procedure shown to be effective for students with moderate-to-severe disabilities (Ault, Wolery, Doyle, & Gast, 1989; Browder, Ahlgrim-Delzell, Spooner,

Mims, & Baker, 2009). During this procedure, when first teaching a skill and giving a direction, the teacher points to the correct response immediately after giving the direction to the student. This is known as the zero time-delay interval. For example, the teacher lays out three cards with the operations "+", "-" and "x". She gives the instructional direction, "Point to plus," and immediately she also points to the card with the plus symbol so the student knows where to point. When the student consistently responds at zero time delay (referred to as Round 1), the teacher gives the direction to the student but delays prompting for the specified number of seconds (e.g., 4-5 seconds) to provide the student with the opportunity to respond independently. This is known as a 5-second delay interval. Teachers should know that the amount of time specified during the delay interval should be individualized for the student. For example, if the student has a physical limitation that requires more time to respond, the number of seconds should be increased to provide the student with enough time to respond. Table 1 presents the time-delay procedure applied to learning numerals.

Several questions should be considered for each student when using this procedure:

- What type of response do you want from the student: Point to the answer (receptive)? Say the answer (expressive)? Pull the answer from a choice board? Eye gaze to an answer?
- To respond, will the student use receptive only or receptive and expressive responding: Point to the answer only? Point to the answer and say it? Use an AAC device to respond?
- If the student requires a prompt, what type of model will you give (e.g., if the student is to say the number while pointing to it, model saying the number while pointing to it: if the student is to point only, model pointing)?
- How many warm-up trials will you give at 0-second time delay?

• How long will you wait before prompting the student for Round 2: 4 seconds? Or work progressively starting at 4

seconds, then 6 seconds, then 8, etc.?

ROUND 1: 0-Second Time Delay	ROUND 2: A-Second Time Delay
ROUND 1. 0-Second Time Delay	ROUND 2. 4-Second Time Delay
Round 1 is a warm-up round. A student (S) may need numerous trials at Round	Round 2 provides the S with the opportunity to respond independently. In
1 before moving to Round 2.	Round 2, give the S up to 4 seconds (or whatever time you select) to respond
Step 1	before giving a prompt.
Present the symbol or word and 3 distractor cards. Review the cards with the S.	Step 1 Present the symbol or word and 3 distractor cards. Review the cards
Step 2	with the S.
In this first round, give the direction to find the symbol or word (e.g., say to the	Step 2
S, Find plus). Provide an immediate prompt (0-second time delay) by pointing	In this second round, give the direction to find the symbol or word (e.g., say to
to the plus sign while giving the direction.	the S, Find plus). Wait 4 seconds for the S to independently respond or to
Step 3	begin to initiate a response. Tell the S to wait if he or she isn't sure, <b>If you are</b>
Provide feedback.	not sure, wait and I will show you.
• If the S points to the correct symbol or word, provide praise (e.g., Yes,	Step 3 Provide feedback.
you found plus).	• If the S points to the correct number, provide praise (e.g., Yes, you
• If the S does not point to the correct response, use a physical prompt to	found plus).
help the S locate the correct symbol or word. Then give praise (e.g.,	• If the S does not point to the correct response, use a physical
Very good. You pointed to the plus).	prompt to help the S locate the correct symbol or word and say, for
	example, <b>This is the plus sign.</b> Then repeat the direction, <b>Find</b>

Step 4 Shuffle the cards between each trial. Repeat for the next symbol or	plus.
word you are focusing on.	<b>Step 4</b> Shuffle the cards between each trial. Repeat for the next symbol or
<b>Step 5</b> Repeat these steps for each S in the group.	word you are focusing on.
Note: There should be no errors in this round unless the S refuses to point or	<b>Step 5</b> Repeat these steps for each S in the group.
cannot imitate pointing. Do 0-second time delay two or three times. When the S	Note: Make sure the student has a neutral hand placement (hands below cards)
consistently respond and/or begins to anticipate the correct response before	prior to asking them to respond. If a student continues to guess, it is important
your controlling prompt, move on to a 4-second time delay (Round 2).	to remind them to ask for help. Use excited praise for an independent correct
	response to encourage responding. If the S misses the response on the 4-second
	time delay round more than twice, return to Round 1 (0-second time delay).

# **Using Least Intrusive Prompts**

The system of least intrusive prompts is another systematic instruction procedure shown to be effective for students with moderate to severe disabilities. This procedure uses a prompting hierarchy to guide students to make a correct response during instruction. First, the teacher provides the student with an opportunity to respond independently. If the student does not make a correct, independent response, or does not respond within a set period of time (e.g., 4-5 seconds), the teacher provides the next least intrusive prompt (e.g., a non-specific verbal prompt). The teacher proceeds through a prompting hierarchy until the student elicits a correct response (Collins, 2007). As shown in the figure, this system places prompts given to students in a hierarchy from the least intrusive to the most

# intrusive.



# The Solutions Project Materials

The Solutions Project is comprised of six units:

- **1. Introductory Unit**
- 2. Group Problem Type
- 3. Compare Problem Type
- 4. Discrimination of Group and Compare Problems
- 5. Change-Addition, Subtraction, and Mixed
- 6. Discrimination of All Problem Types

#### **Lesson Descriptions:**

**Lesson 1 Preteaching/Reviewing Concepts**—The time-delay procedure is used to teach/review key vocabulary, concepts, and symbols. Students also are taught the process of using manipulatives with the graphic organizer to create sets to represent a number sentence and to solve a problem without the context of a word problem. This is to reduce demands on the cognitive load. First, teachers will model how to solve addition or subtraction number sentences on the graphic organizer. Once students have demonstrated an understanding of the process following your model, provide several opportunities for students to try on their own. Provide prompting and feedback, but fade your support to allow students to perform these skills independently.

**Lesson 2 Modeling the Problem-Solving Strategy**—Follow this lesson plan to introduce students to solving targeted problem types using modified schema-based instruction. Students are taught rules with hand motions to help them remember the problem type. Teachers will model the process for solving the word problems step-by-step using a scripted lesson with think alouds. Students will be provided with an opportunity to immediately try each step following the teacher model. Provide coaching and feedback during this lesson, but remember this should be nearly errorless because students are following the teacher model. Repeat this lesson as needed until students understand how to use each step (~2 days). Data of the student's performance is not collected during lessons that model the procedure.

**Lesson 3 Guided Practice**—Once students have general knowledge of how to perform each step of the targeted problem type, follow this lesson plan to give students practice independently solving problems with planned prompting and feedback. Multiple themes are provided. Choose a theme, write the numerals in the problem using the guidelines, provide the required materials, and then follow this lesson plan to give students practice in independently solving the targeted problem type. A mastery target is set for each lesson and students will work toward achieving mastery (~1-2 weeks). If students achieve mastery at different rates, students who have achieved mastery can move on to lessons 4 and 5 to build generalization.

**Lesson 4 Independent Practice with SMARTBoard/Interactive Whiteboard**—After students have used the task analysis and have knowledge of how to perform each step, follow this lesson plan to give them additional practice using the task analysis independently. The SMARTBoard/Interactive Whiteboard template provides practice using additional themes. This format can be done in a small group, taking turns on steps or problems, or individually if students are working at different rates.

Lesson 5 Generalization with Video Simulation Problems—Teachers may elect to create real-world video simulation problems by capturing math story problems in everyday locations on video and showing them to the student. The goal is for students who are able to solve the math story problem using strategies learned in this unit (the target goal). Videos are provided covering topics students may encounter in their everyday lives such as chores at home, pet store, grocery store, thrift store, sporting goods store, and supercenter store. This format can be done in a small group, taking turns on steps or problems, or individually if students are working at different rates.

# **Sample Lesson Themes**

These are sample themes for each unit. (Unit 1 is a review of early numeracy concepts.) You may create your own themes targeting your students' interests.

Theme Sequence						
Unit 2: Group	Unit 3: Compare	Unit 4: Group & Compare Discrimination	Unit 5: Change— Addition	Unit 6 Change— Subtraction	Unit 7: Change— Addition and Subtraction Discrimination	Unit 8: All Problem Type Discrimination
Baseball Game	Department Store	Zoo	Fair	Art Class	Bus	Chores at Home
Beach	Museum	Fast Food	Independence Day	Restaurant	Sporting Goods Store	Flower Shop
Park	Aquarium	Lawn Chores	Classroom Chores	Hardware Store	Gas Station	Airport
Farm	Arcade	Grocery Shopping	Video Game Store	Library	Garden	School Basketball Game
Soccer Game	Salon	Pharmacy	School Dance	Gym Class	Pet Store	Hotel

# **Suggested Teaching Sequence**

Progression of The Solutions Project Across School Year			
	Review	Focus	Dates of Implementation
Intro Lesson Part I		Solidifying early numeracy skills (e.g., rote counting, 1:1 correspondence, making sets) Concepts (e.g., same/different)	Beginning of September
Intro Lesson Part II		Pre-teaching solving addition problems using group graphic organizer	Latter half of September
Group	n/a n/a	Model (2 days) LIP (mastery criteria set at	October October
		8/12 for both problems across 2 consecutive days)	
	n/a	Technology: SMARTBoard	October
Compare	Pre-teach solving subtraction using compare graphic organizer; time delay with words "more/fewer"	Compare Model (2 days)	November
	Review Group problem periodically (independent seat work, 1:1)	LIP	ongoing
		Compare Technology: SMARTBoard	November
		Compare Video Simulation Problems	November
Discrimination Training between Group and Compare	T-chart sorting activity	Model: Present at least 4 problems/day and sort on T- chart (through step 6); vary order of presentation of problem type; solve 2	December
		LIP: 2-4 problems per day	December/January
Change	Pre-teach solving addition problems on change graphic organizer; act out change add verbs	Change-Add Model (2 days) Change-Add LIP (2-4 days)	January
	Pre-teach solving subtraction problems on change graphic organizer;	Change-Subtract Model (2 days) Change-Subtract LIP (2-4	January/February

	act out change subtraction verbs	days)	
	T-chart sorting verbs/actions into addition or subtraction	Change-Mixed Model (2 days) Change-Mixed LIP (1-2 weeks)	February
	Group/Compare (independent seat work, 1:1)	Change-Mixed Technology: SMARTBoard	February
	Group/Compare (independent seat work, 1:1)	Change-Mixed Video Simulation Problems	February
Discrimination of all 3 problem types	n/a	Model think aloud process for selection of problem type with students guiding steps (1 week)	March
	n/a	LIP until mastery criterion is reached (10/12 steps across 4 problem types with 4 discriminations for 2 consecutive days)	March
	n/a	Discrimination SMARTBoard	March/April
		Discrimination Video Simulation Problems	March/April
Fading Supports	n/a	*See Troubleshooting for ideas to fade supports	April
Across Standards	n/a	Model Calculator Use Followed by LIP	April

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# THE SOLUTIONS PROJECT PROGRESS MONITORING AND ASSESSMENT

# How to Conduct the Assessment

Assessments are conducted after the students have completed each unit by mastering at least 10 out of 12 steps with two problems solved for each problem type for two consecutive sessions. To conduct the assessment, have all materials readily available before beginning a testing session with a student. The student will need their set of graphic organizers, checklist, and dry erase marker. Also, have the data sheet for the corresponding unit (see Appendix A).

Assessments either have two (group, compare, and change units), four problems (discrimination between group and compare), or six problems (discrimination of all problem types) depending on the unit. For the group unit, administer at least one problem, but preferably two, so there is an even number across the problem types. When administering the test items, do not prompt or provide any error correction. Reading a step to the student without being requested to do so by the student is considered a prompt. The goal is for independent problem solving.

- Fan the selected problems and allow the student to select a problem. Say to the student, "Solve this problem."
- Wait 15 s for the student to respond.
  - If a student does not initiate a response, try to encourage the student to respond by using a pacing prompt. Examples of pacing prompts include, "How do we get started?" or "Keep going!" or "Show me what to do next!"
  - If the student does not respond or attempt the problem within 15 s, then remove the problem and move to the next problem.
- Record a "+" for an independent correct response, and a "-"for an incorrect response or no response.
  - If the student does not attempt the initial step, "read the problem," all subsequent steps will be marked as "-."
  - If the student attempts some steps, the steps attempted correctly will be marked with a "+" and any step omitted or performed incorrectly will be marked with a "-."
  - If the student does attempt to read a step or the problem and is having difficulty reading or pronouncing the words, you may read the step or the word to help them. This will not be considered a prompt.
- If needed, you may reinforce the student for his or her performance during test items but not for correct responses. Examples of verbal reinforcement are: "You are working so hard! Keep it up!" or "Great job! Keep rocking and rolling!"

# Determining if Students are Ready to Move to the Next Unit

- Two consecutive assessments should be given. The purpose of doing this is not only to look for accuracy, which can be determined with one assessment, but also to look for consistency across a couple of days. Sometimes our students can show mastery one day, but not the next. We want to make sure they truly have mastered the skill prior to moving forward.
- Mastery of a Unit:
  - Two Problem Assessment: When a student can discriminate the problem type by completing steps 4-6 independently AND independently solve the problem for BOTH problems across two assessments; OR
  - Four Problem Assessment: When a student can discriminate both problem types by completing steps 4-6 independently AND independently solve the problem for THREE out of FOUR problems across two assessments; OR
  - Six Problem Assessment: When a student can discriminate the problem type by completing steps 4-6 independently and solve the problem for FIVE out of SIX problems across two assessments, he/she has mastered the curriculum, and you may begin to increase difficulty (see p. 40).

#### TROUBLESHOOTING

If students are struggling to make overall progress, or struggling with individual steps of the task analysis, the following section provides several strategies to try in order to help the students be more successful.

**Prompt Dependency on Reading Steps**. Fade reliance on reading steps to the student after the student has become proficient with the steps of the task analysis. Although the student may ask you to read initially, encourage the student to look at the picture to determine what to do in the step. For example, the student may say, "read the step please" and point to the step, and you respond by pointing to the picture adjacent to the step and say, "what does this picture tell you to do?" If the student can tell you what the step indicates, then he/she no longer needs you to read the step.

**Teaching steps in isolation to mastery.** Sometimes data indicate specific steps of the chained task are causing trouble for students. In this case, teachers should isolate that step and provide multiple opportunities for practice through massed trials to teach the step to mastery before reintroducing back into the chained tasks (Bellamy, Horner, & Inman, 1979). Once students show mastery of the step in isolation, instructors should reintroduce the chain. For example, if a student lacks understanding of the concept of "same" and "different," then when asked if the story problem is about the same thing or different things, s/he will have a very difficult time with this step and may discriminate the problem incorrectly, resulting in an incorrect answer. By taking that step out and teaching the concept of "same" and "different" using pictures and various word problem examples to mastery, and then reintroducing the entire problem solving task analysis, the student is likely to be more successful.

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**Modifying materials.** Slow or no progress also may reflect the need to adapt instructional materials. Oftentimes students with intellectual disabilities have limited fine motor skills or struggle with motor planning, which may impact the format of the graphic organizer or choice of manipulative. Examples of changes to graphic organizers include adding Velcro, increasing their size, and enhancing salient features, such as by highlighting. Based on changes made to the graphic organizer, manipulatives should be chosen that allow for students to demonstrate the most independence possible.

**Tightening prompting.** An additional instructional factor that can influence student progress is prompting and feedback. Consistent and accurate use of the prompting hierarchy by teachers was critical for student success, as was the use of a model-retest for errors. If students showed any regression, teachers increased their behavior-specific praise for steps in which it had previously been faded. Teachers also increased motivation, such as by letting students select what they wanted to work for prior to the work session (e.g., 10 min of iPad time after work).

# **Adapting Materials**

We encourage you to follow the rule "no more different than necessary" with your students. Even when following that principle, some students will need additional modifications to their materials. The materials in *The Solutions Project* curriculum can be adapted for any student's individual response mode. The Student Workbook, all problems, graphic organizers, student checklist, and pictures from the word problems are provided on the CD for convenient printing. The CD also includes a template for creating your own word problems. Pages from the Student Workbook can be enlarged, laminated, Brailled, cut apart and attached to eye gaze boards, or used with augmentative/alternative communication devices.

# Motivation

Motivation is key to getting students to participate in the curriculum. Additional strategies may be needed to increase motivation in your students. Some students may benefit from a token economy, where he or she selects a motivator to work for (e.g., iPad time or snack) and earns a token for each problem completed to acquire the reward.

# **Building Self-Determination Skills**

*The Solutions Project* encourages the development of self-determination skills. The entire curriculum is based on teaching students to self-manage their problem solving skills by following a checklist and checking it off as steps are completed in order to solve a problem. Choice making can easily be embedded into the curriculum as well by allowing students to select the thematic context based on their own interests.

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# **INCREASING DIFFICULTY**

*The Solutions Project* is intended to be the first step in teaching problem solving to students with intellectual disabilities. There are many boundaries on generalization in place in order to help students succeed. If your student has succeeded, and you feel like she or he is ready for an increase in difficulty, there are several ways to boost the curriculum. We do recommend increasing the difficulty in very small steps, such as tackling one of the recommendations at a time, in order for your students to be most successful.

# • Increase the difficulty of numbers used

- Introduce the concept of zero.
- Then introduce double- and triple-digit numerals. We recommend doing these in increments such as numbers 20 and below, then 50, then 100, and then three-digit numbers. Please ensure the context is still relevant. For example, it is unlikely a child will see 60 elephants at the zoo, or purchase 125 apples. With this, students will no longer be able to use manipulatives because it will not be time efficient. Students can write numerals directly on the graphic organizers and use a calculator or other strategies to solve problems with more complex numbers (e.g., addition or subtraction with regrouping).



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- Also, in later elementary school, some students may be ready for the introduction of rational numbers, such as fractions and decimals.
- In compare problems, start to vary the order of the bigger and smaller number so the student must attend to the question and his/her knowledge of larger and smaller numbers in order to find the right answer.
- Fade the supports gradually
  - Fade the pictures above the key nouns for readers, or for students with good listening comprehension skills.
  - Transition from the four-line format to paragraph form to make it look more like a general education problem.
  - Remove the color from the graphic organizers to make them more basic. Transition to outlines so that eventually students will be able to think or draw the borders in order to solve independently without being given a graphic organizer.
- Chunking Steps of the Task Analysis
  - Most students will eventually begin chunking steps of the student checklist together naturally. For example, a student may do steps 1-3 before checking them off, and then 4-6, etc. This is perfectly acceptable and should be encouraged as long as the student continues to follow the steps and get the right solution. For more advanced students, you may try to take away the task analysis, or transition to a mnemonic, like those used in higher incidence literature.

# • Bridge to Algebra

• Until this point, the student has been solving for the missing final amount across all problem types. One way to build difficulty and to better align to grade-level standards in middle school, is to introduce the missing quantity in the middle position and in the initial position. The missing initial position is the most difficult of all. The table below

includes an example of missing quantities in the initial and medial positions across all three problem types.

	Missing initial	Missing medial
group	Doug waited in line to get into the school	The librarian helps students check out books.
	basketball game. There were some men in line.	The librarian helped 42 boys check out books.
	There were 25 women in line. If Jamal saw a	The librarian helped some girls check out books.
	total of 40 people in line, how many men were	If the librarian helped 65 students in all, how many girls did
	in line?	she help?
	Equation: $x + 25 = 40$	Equation: $42 + x = 65$
change	Aaron takes drinks to customers at the	John picked apples at the orchard with his class. He had 8
	restaurant. Aaron had some drinks on his	apples in his bucket and then dropped some. Now he has 4
	tray. Then he spilled 2 drinks. If he has 3	apples left in his bucket. How many apples did he drop?
	drinks left on his tray, how many did he start	
	with?	
compare	Mike bought flowers for his sisters. He bought	Brittany and Hope clipped coupons before they went
	some roses and 4 daisies. If he bought 3 more	shopping. Brittany clipped 7 coupons. Hope clipped 2 fewer
	roses than daisies, how many roses did he buy?	coupons than Brittany. How many coupons did Hope clip?

• It may be helpful to continue keeping the order of information in the problem in the same order that it is used in the

number sentence when bridging to the missing medial and initial positions, especially when first teaching. If students

are ready, then information can be moved around in the problem so it does not match the order in which it is inserted in

the number sentence.

# • Across Standards

• These graphic organizers may be used to solve grade-aligned math problems. For example, students can use the compare graphic organizer to solve problems where the information is posed in a pictograph and students are asked to compare quantities of the data depicted (e.g., How many more students ride the bus to school than arrive by car?)

# • Peer Supports and Inclusion

In the development of *The Solutions Project* curriculum, one study showed that peer tutors could be trained to implement the curriculum with students with moderate intellectual disability (Ley Davis, Spooner, & Saunders, 2017). Peer tutors were trained on solving the targeted problem type and how to provide prompting and feedback.

# • Expanding on the Complexity of The Solutions Project

*The Solutions Project* focuses on a streamlined approach so students with cognitive challenges can experience early success in the new skill of problem solving. This streamlining required placing some parameters on the level of generalization expected. As students experience success, consider adding problems that challenge students to go to the next level of complexity. The following table will describe the parameters in place in the curriculum and provide examples of how to write problems with more variation and how to expand on the complexity.

Parameter of The Solutions	Example	Expanding Level of Complexity
Project		
Numbers are always depicted by numerals.	John bought 3 snickers	John bought three snickers

The order the information is presented	Change:	Change:
in the word problem is the same order	John bought 3 snickers. He ate 1	John ate 1 snicker. He bought 3
that it will be recorded on the graphic	snicker. How many snickers does John	snickers. How many snickers does
organizer and on the number sentence.	have left?	John have left?
	(beginning set, change set, ending set)	(change set, beginning set, ending set)
Group: small group, small group, big group		(
Change: beginning set, change set, ending set		
Compare: bigger number, smaller number		
The missing quantity is always	John bought 3 snickers. He ate 1	John bought 3 snickers. He ate some
presented in the final position. (A+B=?	snicker. How many snickers does John	snickers on the way home. Now he has
or A-B=?)	have left?	2 snickers left. How many snickers did
		John eat?
	3-1=?	3-?=2
No extraneous numerals or information	John bought 3 snickers. He ate 1	John bought 3 snickers and 2 Twix
is included in the word problem outside	snicker How many snickers does John	hars He ate 1 snicker How many
of the anchor sentence.	have left?	snickers does John have left?
In a group problem, when each of the	John bought 3 snickers Bob bought 3	John and Bob each bought 3 snickers
small parts are the same amount the	snickers How many snickers did John	How many snickers did John and Bob
numeral is stated twice.	and Bob buy in all?	buy in all?
Each element of the word problem is	John bought 4 snickers Bob bought 3	John bought 4 snickers and Bob bought
written in a separate sentence	Twix. How many candy bars did John	3 Twix, how many candy bars did they
	and Bob buy in all?	buy in all?
In change problems, a numeral always	John bought 4 snickers. He ate 2	John bought 4 snickers. He ate half of

depicts change amount.	snickers. How many snickers does John have left?	his snickers. How many snickers does John have left?
In change problems, situations where "all" of something was lost do not occur because differences must be of 1 or greater.	John bought 4 snickers. He ate 2 snickers. How many snickers does John have left?	John bought 4 snickers. He ate 4 snickers. How many snickers does John have left?
In change problems, zero is not used in the change amount because it does not indicate a change, therefore situations where "nothing" was gained or lost does not occur.	John bought 4 snickers. Bob gave him 2 more snickers. How many snickers does John have now?	John bought 4 snickers. Bob did not give him any snickers. How many snickers does John have now?
In change-subtract and compare problems, the greater amount is presented first in the word problem to set the student up for a subtraction number sentence.	John bought 4 snickers. Bob bought 3 snickers. How many more snickers did John buy than Bob?	John bought 3 snickers. Bob bought 4 snickers. How many more snickers did Bob buy than John?
Unknown information is always to the right of the equal sign.	$\mathbf{A} + \mathbf{B} = ?$	$? = \mathbf{A} + \mathbf{B}$

# Appendix A Pre-Screening Tool Data Sheet

Student/Teacher:		Date:					
Researcher:		Video:		_			
Skill		Points	C/I	Notes			
Number ID	Receptively ID numbers	1	+ -	1 2 3 4 5 6 7 8 9 10			
Number ID	Verbally ID numbers*	1	+ -	1 2 3 4 5 6 7 8 9 10			
More/less	More Fewer Expressive ID Fewer Expressive ID More	4	+ - + - + - + -				
Same/Different	Different (picture) Same (picture) Different (WP) Same (WP)	4	+ - + - + - + -				
Count manipulatives (scattered, moveable)	3 9	2	+ - + -				
Count pennies (nonmoveable pictures)	4 10	2	+ - + -				
Making sets	5 7 10	3	+ - + - + -				
Reading equation	Addition Subtraction	2	+ - + -	Make note of ANY part missed. 4 + 1 = 5 4 - 1 = 3			
Adding with sets	3+4= 7	1	+ -	Note if solved using different method (e.g., TouchMath, mental math, finger counting, tallies). Did student perform any part of this skill?			
Subtracting with sets	7-5 = 2	1	+ -	Note if solved using different method (e.g., TouchMath, mental math, finger counting, tallies). Did student perform any part of this skill?			
Identifying signs	Show me + Show me – Draw +	4	+ - + - + -				

	Draw –		+	-	
Calavilaten	Chauring a l				
Calculator	show me +	5	+	-	
	Show me –		+	-	
	Show me =		+	-	
	Solve +		+	-	
	Solve -		+	-	
Solve	Group	3	+	-	
	Change		+	-	
	Compare		+	-	
	1				1

Score: \_\_\_\_\_

\*If student has difficulty with all of the numbers on page, present on flashcards in random order. Also, if student inverses number (e.g., switches 9 and 6), present on flashcards in random order.

	Date/Student SMARTBoard (SB) or Paper-										
	and-Pencil (PP)										<u> </u>
Step	S Response										
12. Write answer	Writes/Selects numeral	12	12	12	12	12	12	12	12	12	12
11. Solve	Combines sets and counts total to add	11	11	11	11	11	11	11	11	11	11
10. Make sets	Creates sets in small circles	10	10	10	10	10	10	10	10	10	10
9. + or -	Writes/selects "+" in # sentence	9	9	9	9	9	9	9	9	9	9
8. Fill-in number	Writes/selects numerals in #	8	8	8	8	8	8	8	8	8	8
sentence	sentence										
7. Circle the numbers	Circles #s in word problem	7	7	7	7	7	7	7	7	7	7
6. Choose GO	Selects Group GO	6	6	6	6	6	6	6	6	6	6
5. Use my rule	States rule	5	5	5	5	5	5	5	5	5	5
4. Same, Different, More/Fewer	Circles different on GO	4	4	4	4	4	4	4	4	4	4
3. Find label in question	Underlines label	3	3	3	3	3	3	3	3	3	3
2. Circle the "what"	Circles nouns	2	2	2	2	2	2	2	2	2	2
1. Read the problem	S has T read problem or S reads problem	1	1	1	1	1	1	1	1	1	1
Notes in space below	TOTAL CORRECT:										
Scoring Procedures											
-Mark independent ar -Circle total number o	nswers with / f independent correct answers										

# **Group Word Problems: Student Data Sheet**

	Date/Student										
	SMARTBoard (SB) or Paper-										
	and-Pencil (PP)										
Step	S Response										
12. Write answer	Writes/Selects numeral	12	12	12	12	12	12	12	12	12	12
11. Solve	Finds difference between sets and moves to blue difference circle; counts amount	11	11	11	11	11	11	11	11	11	11
10. Make sets	Creates sets in green and red arrays	10	10	10	10	10	10	10	10	10	10
9. + or -	Writes/selects "-" in # sentence	9	9	9	9	9	9	9	9	9	9
8. Fill-in number	Writes/selects numerals in #	8	8	8	8	8	8	8	8	8	8
sentence	sentence	L									
7. Circle the numbers	Circles #s in word problem	7	7	7	7	7	7	7	7	7	7
6. Choose GO	Selects correct GO	6	6	6	6	6	6	6	6	6	6
5. Use my rule	States rule	5	5	5	5	5	5	5	5	5	5
4. Same, Different, More/Fewer	Circles more or fewer on GO depending on label in WP	4	4	4	4	4	4	4	4	4	4
3. Find label in question	Underlines label; note the label should include "more/fewer"	3	3	3	3	3	3	3	3	3	3
2. Circle the "what"	Circles nouns	2	2	2	2	2	2	2	2	2	2
1. Read the problem	S has T read problem or S reads problem	1	1	1	1	1	1	1	1	1	1
Notes in space below	TOTAL CORRECT:										
Scoring Procedures	·		-				•			-	
-Mark independent ar	iswers with /										
-Circle total number of	f independent correct answers										

**Compare Word Problems: Student Data Sheet** 

	Date/Student									
	Problem Type Code: Group (G), Compare More (CM), Compare Fewer (CF) *Aim for 3 per session									
Step	S Response									
12. Write answer	Writes/Selects numeral	12	12	12	12	12	12	12	12	12
11. Solve	Moves final amount and counts	11	11	11	11	11	11	11	11	11
10. Make sets	Makes sets according to problem type	10	10	10	10	10	10	10	10	10
9. + or -	Writes/selects "+" or "-" in # sentence	9	9	9	9	9	9	9	9	9
8. Fill-in number sentence	Writes/selects numerals in # sentence	8	8	8	8	8	8	8	8	8
7. Circle the numbers	Circles #s in word problem	7	7	7	7	7	7	7	7	7
6. Choose GO	Selects correct GO	6	6	6	6	6	6	6	6	6
5. Use my rule	States rule	5	5	5	5	5	5	5	5	5
4. Same, Different, More/Fewer	Circles correct response on GO	4	4	4	4	4	4	4	4	4
3. Find label in question	Underlines label	3	3	3	3	3	3	3	3	3
2. Circle the "what"	Circles nouns	2	2	2	2	2	2	2	2	2
1. Read the problem	S has T read problem or S reads problem	1	1	1	1	1	1	1	1	1
Notes in space below	TOTAL CORRECT:									

**Discrimination of Group/Compare Problems: Student Data Sheet** 

Scoring Procedures

-Mark independent answers with /

-Circle total number of independent correct answers

	Date/Student SMARTBoard (SB) or Paper-										
	and-Pencil (PP)										
Step	S Response										
12. Write answer	Writes/Selects numeral	12	12	12	12	12	12	12	12	12	12
11. Solve	Moves remaining set to ending amount blue square; counts	11	11	11	11	11	11	11	11	11	11
10. Make sets	Creates initial set and then change1010101010set (either adding more or moving some from initial set to trash can)10101010					10	10	10	10		
9. + or -	Writes/selects "+" or "-" in # sentence	9	9	9	9	9	9	9	9	9	9
8. Fill-in number	Writes/selects numerals in #	8	8	8	8	8	8	8	8	8	8
sentence	sentence	ļ	Ļ						Ļ		
7. Circle the numbers	Circles #s in word problem	7	7	7	7	7	7	7	7	7	7
6. Choose GO	Selects correct GO	6	6	6	6	6	6	6	6	6	6
5. Use my rule	States rule	5	5	5	5	5	5	5	5	5	5
4. Same, Different, More/Fewer	Circles same on GO	4	4	4	4	4	4	4	4	4	4
3. Find label in question	Underlines label	3	3	3	3	3	3	3	3	3	3
2. Circle the "what"	Circles nouns	2	2	2	2	2	2	2	2	2	2
1. Read the problem	S has T read problem or S reads problem	1	1	1	1	1	1	1	1	1	1
Notes in space below	TOTAL CORRECT:										
Scoring Procedures			<u>.</u>	<u>.</u>	<u>.</u>			<u>.</u>			
-Mark independent ar	iswers with /										
-Circle total number of	f independent correct answers										

Change Word Problems: Student Data Sheet

	Date/Student										
	Problem Type Code: Group (G), Change Add (CA), Change Subtract (CS), Compare More (CM), Compare Fewer (CF) *Aim for 5-6 problems per session, 2 of each type										
Step	S Response										
12. Write answer	Writes/Selects numeral	12	12	12	12	12	12	12	12	12	12
11. Solve	Moves final amount and counts	11	11	11	11	11	11	11	11	11	11
10. Make sets	Makes sets according to problem type	10	10	10	10	10	10	10	10	10	10
9. + or -	Writes/selects "+" or "-" in # sentence	9	9	9	9	9	9	9	9	9	9
8. Fill-in number sentence	Writes/selects numerals in # sentence	8	8	8	8	8	8	8	8	8	8
7. Circle the numbers	Circles #s in word problem	7	7	7	7	7	7	7	7	7	7
6. Choose GO	Selects correct GO	6	6	6	6	6	6	6	6	6	6
5. Use my rule	States rule	5	5	5	5	5	5	5	5	5	5
4. Same, Different, More/Fewer	Circles correct response on GO	4	4	4	4	4	4	4	4	4	4
3. Find label in question	Underlines label	3	3	3	3	3	3	3	3	3	3
2. Circle the "what"	Circles nouns	2	2	2	2	2	2	2	2	2	2
1. Read the problem	S has T read problem or S reads problem	1	1	1	1	1	1	1	1	1	1
Notes in space below	TOTAL CORRECT:										

# **Discrimination of All Word Problems: Student Data Sheet**

1.		Read the problem
2.	horses	Circle the "whats"
3.	How many?	Write the label
4.	or or or Fewer	Same? Different? More/fewer?
5.		Choose the graphic organizer
6.		Say the rule
7.	3	Circle the numbers
8.		Fill-in the number sentence
9.	Number Sentence:	Write + or –
10.		Make sets
11.		Solve and write answer



Department of Education.

# "BIGGER number, SMALLER number, DIFFERENCE between the two"





fewer/less

DIFFERENCE

more

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# "One thing, SAME, add more or take away, CHANGE"



